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FOCUSING AND SCATTERING OF PLANE SHOCK WAVES AT AN INTERFACE BETWEEN ANISOTROPIC ELASTIC MEDIA

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The paper is focused on the problem of constructing evolving fronts of quasilongitudinal and quasitransverse shock waves formed by incidence of an initial plane shock wave on a curvilinear interface between elastic transverse isotropic media with different physical properties. The parameter continuation method and the Newton algorithm are used to solve nonlinear Snell's equations. A method for calculating discontinuities of field functions is proposed. Shock-wave scattering and focusing as a particular case of bifurcation of shock fronts and formation of caustics are considered. A numerical example is given.

Introduction. Geometrical optics considers focusing of light rays (light waves) by optical devices — lenses and mirrors, which are widely used to increase locally illumination and concentration of light (heat) energy. Similar phenomena are observed during propagation of electromagnetic waves. Methods of geometrical optics [1–3] are typically used to describe numerically the focusing and scattering of light rays and to study their special features.

The special features of reorganization of phase fronts and caustic surfaces studied in [1, 4, 5] are used to analyze physical phenomena not only in geometrical optics but also in acoustic studies and radiophysics. These features can be found even in the simplest optical systems consisting of homogeneous media separated by a curvilinear boundary. To describe such systems, it suffices to use two consequences of Fermat's least time principle: 1) for reflection of light from a reflecting surface, the incidence angle is equal to the reflection angle; 2) when light passes through an interface, both the incidence and refraction angles obey the refraction law. In this case, since light rays are focused at the envelope of these rays (caustic), the light-field intensity along the caustics increases unlimitedly and singularities are formed at the phase front. During motion of the front surface, these singularities slide along the caustics and rearrange at certain moments when singularities occur on the caustics, too. The structure of singularities of phase fronts is more complex than the structure of singularities of the corresponding caustics because the codimension of the phase fronts exceeds by unity that of the caustics.

Since shock waves in elastic media are of the same nature as light and electromagnetic waves, their interaction with the curvilinear surfaces of inhomogeneities of elastic media is also accompanied by changes in the direction of the waves, which leads to their focusing or scattering.

As a rule, in studies of the singularities of shock-wave fronts in elastic media, geometrical constructions of moving discontinuity surfaces of field functions and calculations of discontinuities are of special interest. These data provide important information on shock-wave fronts and intensity of the main part of the impulse transferred by the wave at any point of its front. To formulate and solve these problems, the theory of elasticity uses methods of geometrical optics, in particular, the zero approximation of the ray method, which describes numerically a wide range of wave phenomena of different physical natures [3, 6–8]. The ray method allows one to determine the optical path function of a wave (eikonal) and, using the eikonal equation, construct a system of rays and fronts of the shock wave. We note that even for isotropic elastic media, for which this problem is easy to solve, it is rather problematic to study wave interactions with an interface between media with different mechanical properties (lenses, etc.). These interactions cause formation of caustics due to energy focusing and an unlimited increase in field intensity.

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However, the study of shock-wave focusing by anisotropic "lenses" and "mirrors" is more laborious for the following reasons: in anisotropic systems, field functions are vector functions, the physical picture is more complicated because for each direction there are three types of waves with different polarizations, the phase velocities of waves depend on the direction of their propagation, rays are generally nonorthogonal to the shock-wave front surface, and radial velocities are different from phase velocities and their directions are not necessarily in univocal correspondence. Wave diffraction at the interface is also more complicated because the corresponding Snell's relations become substantially nonlinear due to unknown phase velocities of reflected and refracted waves. Therefore, to determine directions of rays reflected from the interfaces, we have to solve systems of nonlinear equations. By virtue of nonuniqueness of solutions of these systems, caustics can form even when a regular shock wave is incident on an interface of small curvature (unlike in homogeneous isotropic media), which leads to a wider variety of diffraction processes.

Podil'chuk and Rubtsov [3] and Anik'ev et al. [8] described methods for studying shock-wave diffraction at an interface between isotropic elastic media, and Gulyaev et al. [7] considered interaction of a shock wave with a plane interface between anisotropic media. In this paper, we study special features of wave transformation at curvilinear interfaces between anisotropic media.

The problems of interaction of an incident wave with an interface between anisotropic media has been commonly solved by constructing curved refraction vectors [2, 6] (graphic method). In the present paper, we use the parameter continuation method [9], which allows easy identification of bifurcation states.

Formulation of the Problem. In Cartesian coordinates (x_1, x_2, x_3), the motion of particles of an elastic medium is defined by the system of differential equations

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} \frac{\partial^2 u_q}{\partial x_k \, \partial x_p} - \frac{\partial^2 u_i}{\partial t^2} = 0 \qquad (i = 1, 2, 3), \tag{1}$$

where $\lambda_{ik,pq} = c_{ik,pq}/\rho$ ($c_{ik,pq}$ are elastic parameters and ρ is the density), u_1, u_2, and u_3 are components of the elastic displacement vector, and t is time.

In anisotropic media, the rays are generally nonorthogonal to the surfaces of shock-wave fronts, and, therefore, we differentiate between the phase ($\boldsymbol{v}$) and ray ($\boldsymbol{\xi}$) velocities, assuming that the front is a continuous phase surface: $\boldsymbol{n} \cdot \boldsymbol{r} - vt =$ const and each elementary surface of the front moves along the unit normal to this surface $\boldsymbol{n}$ at velocity v. Here $\boldsymbol{r}$ is the radius-vector of a point of the front.

The wave polarization vector $\boldsymbol{A}$ and its phase velocity $\boldsymbol{v}$ for the specified direction $\boldsymbol{n}$ can be constructed as proper numbers and vectors of matrix coefficients of the homogeneous system of algebraic equations [2, 6]

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} n_k n_p A_q - v^2 A_i = 0 \qquad (i = 1, 2, 3). \tag{2}$$

The condition of existence of nontrivial solutions of this system leads to the following third-order equation with respect to the squared phase velocity v^2:

$$\Big| \sum_{k,p=1}^{3} \lambda_{ik,pq} n_k n_p - v^2 \delta_{iq} \Big| = 0.$$

Using this equation, for each direction of the normal $\boldsymbol{n}$, we can determine the velocities of three differently polarized waves and arrange them in descending order: $v_1^2(\boldsymbol{n}) > v_2^2(\boldsymbol{n}) \geqslant v_3^2(\boldsymbol{n}) > 0$.

Assuming that the value of v^2 in system (2) is equal to one of the values of $v_r^2(\boldsymbol{n})$ $(r = 1, 2, 3)$, we obtain systems of equations for components of the polarization vectors $\boldsymbol{A}^{(r)}$ of the three waves moving in the direction considered $\boldsymbol{n}$ with their phase velocities $\boldsymbol{v}_r(\boldsymbol{n})$:

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} n_k n_p A_q^{(r)} - v_r^2 A_i^{(r)} = 0 \qquad (i = 1, 2, 3).$$

For all $\boldsymbol{n}$, the polarization vectors obey the orthogonality conditions $\boldsymbol{A}^{(i)}(\boldsymbol{n}) \cdot \boldsymbol{A}^{(k)}(\boldsymbol{n}) = \delta_{ik}$ $(i, k = 1, 2, 3)$.

The surface of the shock-wave front is defined by the relation

$$\tau\,(x_1, x_2, x_3) - t = 0, \tag{3}$$

in which the function τ must satisfy the differential first-order partial equation [2]

$$\sum_{i,k,p,q=1}^{3} \lambda_{ik,pq} p_k p_p A_q^{(r)} A_i^{(r)} = 1. \tag{4}$$

This equation extends the eikonal equation in geometrical optics to elastic anisotropic waves.

The quantities p_k ($k = 1, 2, 3$) included in (4) are components of the refraction vector $p_k \equiv \partial\tau/\partial x_k = n_k/v_r(\boldsymbol{n})$ ($k = 1, 2, 3$).

To construct the shock-wave front (3) in a homogeneous anisotropic medium ($\rho = \text{const}$), it is necessary to find solutions of Eq. (4), which reduces to the following system of ordinary differential equations by using the method of characteristics:

$$\frac{dx_k}{d\tau} = \xi_k = \sum_{i,p,q=1}^{3} \lambda_{ik,pq} p_p A_q^{(r)} A_i^{(r)}, \qquad \frac{dp_k}{d\tau} = 0 \quad (k = 1, 2, 3). \tag{5}$$

The first group of these equations describes radial wave propagation at radial velocity $\boldsymbol{\xi} = \boldsymbol{\xi}^{(r)}(\boldsymbol{n}, x_k)$. From the second group of equations, it follows that in a homogeneous medium, the rays are straight lines.

The system of rays and fronts constructed using Eq. (5) is used to determine wave intensities in the vicinity of the front. For this, it is convenient to use the radial series expansion of solution (1):

$$u_q = \sum_{m=0}^{\infty} u_q^{(m)}(x_1, x_2, x_3) f_m[t - \tau\,(x_1, x_2, x_3)] \qquad (q = 1, 2, 3), \tag{6}$$

where it is assumed that the functions f_m corresponding to the relations $f_m'(y) = f_{m-1}(y)$ have discontinuities in derivatives, for example, of order $n + 2$ [2].

To study the behavior of a shock wave in a small neighborhood of the front, it suffices to retain only one term $m = 0$ in expansion (6) and calculate the wave-field intensity vector $\boldsymbol{U}^{(0)}$ from the homogeneous system of equations

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} p_k p_p U_q^{(0)} - U_i^{(0)} = 0 \qquad (i = 1, 2, 3),$$

whose solution in radial coordinates (τ, α, β) is written as follows [2]:

$$U_q^{(0)} = c_0(\alpha, \beta) A_q^{(r)}(\tau, \alpha, \beta) f_0[t - \tau(x_1, x_2, x_3)]/\sqrt{J(\tau, \alpha, \beta)} \qquad (q = 1, 2, 3).$$

Here $J = \partial(x_1, x_2, x_3)/\partial(\tau, \alpha, \beta)$ is the functional determinant of conversion of the radial coordinates to Cartesian coordinates.

The relations considered above allow us to construct a family of rectilinear rays and sequences of shock-wave fronts in a homogeneous anisotropic medium and to calculate the discontinuity of the field functions on the front surface during the evolution of this surface.

Method of Solution. We consider two transverse isotropic media whose elastic parameters have symmetry axes coincident with the axis Ox_2 of Cartesian coordinates. By virtue of symmetry, the components $c_{ik,pq}$ of the elastic-constant tensor for each medium can be conveniently written as a 6×6 square matrix $C_{\alpha\beta}$ whose elements are put in correspondence by the scheme

$$(11) \leftrightarrow 1, \qquad (22) \leftrightarrow 2, \qquad (33) \leftrightarrow 3, \qquad (23) = (32) \leftrightarrow 4, \tag{7}$$

$$(31) = (13) \leftrightarrow 5, \qquad (12) = (21) \leftrightarrow 6.$$

Since the elastic properties of a transverse isotropic medium are characterized by five irreducible parameters, the matrix $C_{\alpha\beta}$ can be written as follows [9]:

$$C_{\alpha\beta} = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda - l & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu - p & \lambda - l & 0 & 0 & 0 \\ \lambda - l & \lambda - l & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu - m & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu - m \end{pmatrix}.$$

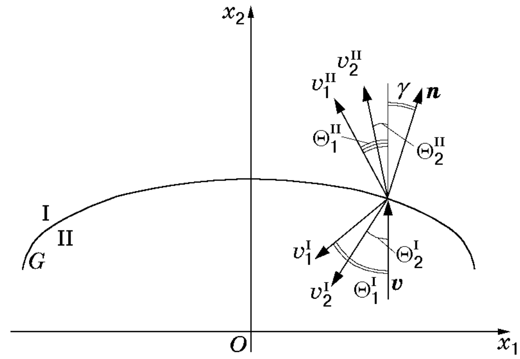

Fig. 1. Orientation of the phase velocities of shock-wave fronts.

Here λ and μ are Lamé parameters and l, m, and p are parameters that describe properties of the transverse isotropic medium.

The main diagonal minors of the matrix (7) satisfy the conditions

$$c_{11} > 0, \qquad \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} > 0, \qquad \ldots, \qquad |c_{\alpha\beta}| > 0$$

which provide for positive definiteness of the corresponding squares.

Let a plane quasilongitudinal shock wave be generated in one of the media considered (media I). The polarization vector of this wave is directed along the axis Ox_2 (by virtue of the symmetry conditions, the wave is purely longitudinal). We study the diffraction of this wave upon its interaction with a curvilinear axisymmetric interface G, whose symmetry axis also coincides with the axis Ox_2. Since the problem formulated is axisymmetric, it suffices to consider the reorganization and formation of shock-wave tracks on one of the planes, for example, on the plane $x_3 = 0$, which contains the axis of symmetry. We adopt the "local–plane approximation" [2], according to which in the place of wave incidence on the elementary surface separating the surfaces G in the incidence plane $x_3 = 0$, all refracted and reflected waves belong to this plane, i.e., the third components of all polarization vectors are zero. This allows us to consider angles Θ_ν^{I} and Θ_μ^{II} $(\mu, \nu = 1, 2)$ for the wave refracted and reflected from the interface G and use the generalized Snell's law expressed by the equalities [2, 6]

$$(1/v)\sin\gamma = \sin(\Theta_\nu^{\mathrm{I}} + \gamma)/v_\nu(\Theta_\nu^{\mathrm{I}}) = \sin(\Theta_\mu^{\mathrm{II}} - \gamma)/v_\mu(\Theta_\mu^{\mathrm{II}}) \qquad (\nu, \mu = 1, 2), \tag{8}$$

where γ is the angle between the direction Ox_2 and the normal to the interface G at the point of ray incidence, Θ_ν^{I} and Θ_μ^{II} $(\nu, \mu = 1, 2)$ are angles between the wave normal and the axis Ox_2 of the waves reflected into medium I and transmitted into medium II, respectively (Fig. 1). The subscripts $\nu = \mu = 1$ correspond to quasilongitudinal waves qP and the subscripts $\nu = \mu = 2$ refer to quasitransverse waves qS; the subscripts minus and plus refer to wave parameters before and after reorganization at the surface G, respectively, and the superscripts I and II refer to parameters of the reflected and refracted waves, respectively. After interaction of the wave front P_- with the axisymmetric surface G, the rays of the reflected waves qP_+^{I} and qS_+^{I} and refracted waves qP_+^{II} and qS_+^{II} are inclined to the axis Ox_2 at certain angles because the transverse isotropy of media I and II begin to manifest itself by disturbance of the longitudinal orientation of the polarization vectors of the shock waves qP_+^{I} and qP_+^{II}.

The difference between relations (8) from the Snell's relations for isotropic media is due to the dependence of v_ν and v_μ on the corresponding angles Θ_ν^{I} and Θ_μ^{II} and to the implicit dependence on the angle γ. The angles Θ_ν^{I} and Θ_μ^{II} $(\nu, \mu = 1, 2)$ of ray reflection and refraction at a certain point of the interface G are determined by solving the nonlinear equations (8) by the Newton method combined with the parameter continuation method [9]. It is convenient to choose the angle γ as the parameter. For example, for the first equation in (8) for a known state $\gamma = \gamma^i$, $(\Theta_\nu^{\mathrm{I}})^i$, a small increment of the parameter $\Delta\gamma^i$ corresponds to increments of the directing angles of phase velocities of reflected waves:

$$(\Delta\Theta_\nu^{\mathrm{I}})^i = \frac{v_\nu^i(\Theta_\nu^{\mathrm{I}})\cos\gamma^i - v\cos(\Theta_\nu^{\mathrm{I}} + \gamma)^i}{v\cos(\Theta_\nu^{\mathrm{I}} + \gamma)^i - \partial v_\nu^i(\Theta_\nu^{\mathrm{I}})/\partial\Theta_\nu \sin\gamma^i}\,\Delta\gamma^i + \delta^i. \tag{9}$$

Here $\delta^i = v\sin(\Theta_\nu^{\mathrm{I}} + \gamma)^i - v_\nu^i(\Theta_\nu^{\mathrm{I}})\sin\gamma^i$ are the residuals at this stage of the solution.

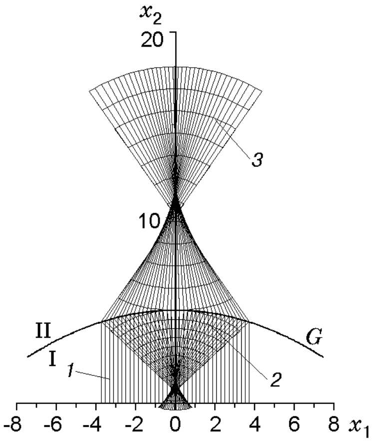

Fig. 2

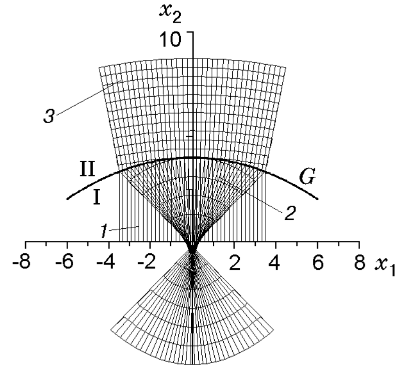

Fig. 3

Fig. 2. Focusing of reflected and refracted quasilongitudinal shock waves: 1) P_-; 2) $qP_+^{\rm I}$; 3) $qP_+^{\rm II}$.
Fig. 3. Focusing of reflected quasilongitudinal shock wave (notation same as in Fig. 2).

Calculations by scheme (9) are possible if there exists a certain initial state γ^0, v^0, $(\Theta_\nu+\gamma)^0$, $v^0_\nu(\Theta_\nu)$. In the case of an axisymmetric interface between media, it is convenient to choose $\gamma^0 = 0$, i.e., to begin constructing the family of incident, reflected, and refracted rays with a ray oriented along the axis Ox_2. Using formula (9), we obtain the unique increment $\Delta\Theta^{\rm I}_\nu$ for the incidence angle γ^i, for which the denominator on the right side of (9) is nonzero. Therefore, the equalities

$$v\cos{(\Theta^{\rm I}_\nu+\gamma)^i} - \frac{\partial v^i_\nu(\Theta^{\rm I}_\nu)}{\partial\Theta^{\rm I}_\nu}\sin\gamma^i = 0 \qquad (\nu = 1, 2) \tag{10}$$

are bifurcation conditions of the solution. To continue this solution through this state, it is necessary to add terms of the second order (or, if necessary, of the third and higher orders) to (9) [10].

The condition of possible nonuniqueness (10) for solutions of system (8) corresponds to convergence (tangency) and intersection of reflected and refracted rays after interaction of incident rays with the interface G. A set of such critical situations is due to the formation of an envelope of the ray family, i.e., a caustic. In this case, caustics can cause formation of geometrical singularities on the surfaces of reflected and refracted wave fronts as a result of interaction of a regular incident wave front even with a plane interface G between anisotropic media.

Since singularities of a shock front are formed on caustics, focusing of this front also occurs on the caustics, accompanied by an unlimited increase in field intensity at places with geometrical singularities.

Numerical Realization and Analysis of the Results. Using the algorithm proposed, we solved the problem of diffraction of the front of a plane longitudinal shock wave generated in medium I on a parabolic interface G between two transverse isotropic elastic media. This wave is polarized along the symmetry axis of the elastic properties of both media, which coincides with the axis of rotation of the interface G.

Since the problem is axisymmetric, after interaction of the incident front with the interface G, the intensity of refracted and reflected quasitransverse waves polarized orthogonally to the plane containing the symmetry axis is equal to zero. Therefore, the plane front incident on the interface G generates only two types of axisymmetric reflected and refracted waves polarized in the axial-section plane.

The mechanical constants were assumed to be close to the constants of dolomite for medium I ($\lambda_1 = 4.971\cdot 10^{10}$ Pa, $\mu_1 = 3.912\cdot 10^{10}$ Pa, and $\rho_1 = 2.650\cdot 10^3$ kg/m^3) and sandstone for medium II ($\lambda_2 = 3.413\cdot 10^9$ Pa, $\mu_2 = 1.361\cdot 10^{10}$ Pa, and $\rho_2 = 2.760\cdot 10^3$ kg/m^3). The quantities l, m, and p, violating isotropic properties, were varied, and calculations were performed for various combinations of their values. The shape of the interface between the two transverse isotropic elastic media was assumed to be parabolic. This interface intersects the axial plane along the curve $x_1 = 5 - 0.055x_2^2$. Figure 2 shows focusing of a plane shock wave by a concave interface between media characterized by anisotropic parameters $l_i = 0.1\lambda_i$, $m_i = 0.2\mu_i$, and $p_i = 0.1(\lambda_i + 2\mu_i)$ ($i = 1, 2$). One can see the radial system of the incident longitudinal wave P_- (curves 1) and the quasilongitudinal wave $qP_+^{\rm I}$ reflected into medium I (curves 2) and quasilongitudinal wave $qP_+^{\rm II}$ refracted into medium II (curves 3)

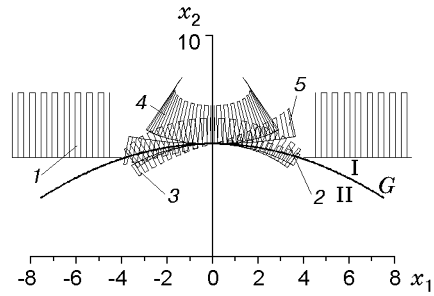

Fig. 4. Diagrams of displacement velocities at the fronts of reflected and refracted quasilongitudinal and quasitransverse shock waves: 1) P_-; 2) $qP_+^{\rm I}$; 3) $qS_+^{\rm I}$; 4) $qP_+^{\rm II}$; 5) $qS_+^{\rm II}$.

(quasitransverse waves $qS_+^{\rm I}$ and $qS_+^{\rm II}$ formed upon such diffraction are less intense and not shown in Fig. 2). Figure 2 also shows axisymmetric fronts of the incident longitudinal waves and those of the reflected and refracted quasilongitudinal shock waves on a plane containing the symmetry axis of the system considered. In the areas of ray concentration, the stress intensity increases. Obviously, in the model used here, the wave-field intensity increases unlimitedly at the sites of focusing of the reflected and refracted waves. The presence of focusing zones on the symmetry axis of the problem for reflected and refracted rays and the focal distance are determined by the surface geometry of the interface G and the relation between the elastic properties of the media. Here the term of "focal distance" is used conditionally because the ray focusing is generally not precise.

When a plane shock wave from a less "optically dense" medium (sandstone) is incident on a concave interface of the same geometry that separates this medium from a more "optically dense" medium (dolomite), the refracted rays of the quasilongitudinal wave $qP_+^{\rm II}$ are scattered, and the reflected rays of this wave $qP_+^{\rm I}$ are focused. This phenomenon is shown in Fig. 3 for elastic media with the properties indicated above.

Intensities of the shock waves generated by interaction of a plane shock wave of unit intensity with a concave parabolic interface G between transverse isotropic elastic media (all parameters of the media correspond to the case shown in Fig. 2) are plotted in Fig. 4 as profiles at the corresponding fronts at the same time. As can be seen, incidence of the plane wave P_- (curves 1) initiates the reflected quasilongitudinal wave $qP_+^{\rm I}$ (curves 2) and quasitransverse wave $qS_+^{\rm I}$ (curves 3) and the refracted quasilongitudinal wave $qP_+^{\rm II}$ (curves 4) and quasitransverse wave $qS_+^{\rm II}$ (curves 5), which have curvilinear axisymmetric fronts with nonuniform intensity distribution. In this case, the fronts of the quasitransverse waves are always behind the fronts of the quasilongitudinal waves, and the diagrams of wave intensities for the former have a skew-symmetric shape. At the sites of ray focusing and on the caustics, wave field intensities can far exceed incident-wave intensity.

For incidence of a plane shock wave on a concave parabolic interface between elastic transverse isotropic media I and II, the reflection and refraction process changes quantitatively. On a concave interface, the rays are focused and scattered; in contrast, on a convex interface, they are scattered and then focused. We note that for convex interfaces, the rays of the reflected quasitransverse waves $qP_+^{\rm I}$ are always divergent.

Conclusions. The problem of shock-wave propagation in transverse isotropic elastic media is considered. The most complex transformations of these waves occur at the interfaces between elastic media with different mechanical properties, where an incident wave generates two triples (for axisymmetric problems, two couples) of refracted and reflected waves polarized differently. Because these waves are described by discontinuous functions and change dramatically with time and in space, description (in the class of special functions) of these waves by analytical and numerical methods is difficult.

A particular problem of reorganization of a plane shock-wave front at a parabolic interface between two different transverse isotropic media was solved using methods of geometrical optics, the zero approximation of the ray method, the parameter continuation method, and the Newton method. Special features of the formation of caustic surfaces and focusing of reflected and refracted rays were studied. A sharp increase in shock-wave intensities at the sites of ray focusing and formation of caustics was described numerically.

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